# STATIC ANALYSIS OF REINFORCED CONCRETE BEAMS STRENGTHENED WITH CFRP COMPOSITES

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This paper presents the possibility to apply numerical simulation in static analysis of reinforced concrete structure strengthened with carbon fibre reinforced polymer composite strips (CFRP). Reinforced concrete beams, with strengthening in form values CFRP made of carbon fibres and epoxy resin, featuring various width, as well as non-strengthened bent beams, were analysed. The simply supported beams arranged in a free support scheme were subjected to two concentrated forces within full range of loading (until collapse). The numerical analysis was performed through application of the Finite Elements Method (FEM), and the calculation model applied took into account the geometric and physical nonlinearity. The problem was solved by application of the *quasi-static* strategy method of calculations using ABAQUS software. While analysing the results, we focused on the run of changes in structure displacement and development of material damage, up to the point of destruction of the beam.

*Key words:* reinforced concrete structures, strengthening, composite strips, Finite Elements Method.

## 1. Introduction

The development of composite materials is the reason that more and more frequently structures made of traditional and composite materials become solutions for engineering problems pertaining to, among other things, strengthening of building structures [1]. A method that is frequently used for this purpose is application of CFRP strips made of carbon fibres and epoxy resin. Increase of interest in such composite materials originates from their extensive usefulness. They feature, among other, several times higher tensile and fatigue strength, compared to structural steel, associated with good chemical resistance, low unit weight, ease easy strengthening and modest outlays on maintenance of composite structure elements. The modulus of CFRP elasticity falls within the range from 150 GPa to 270 GPa, which allows the selection of strips of proper elasticity depending on the cross-section operating conditions.

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Any experimental research of reinforced concrete elements strengthened with CFRP is labour consuming and expensive. Therefore, it is worth applying opportunities for numerical analysis of such structures in research by using, among other, the Finite Elements Method (FEM) [2, 3]. The numerical analysis allows to obtain results that cannot be recorded with measuring devices; it provides extra information about the investigated process and allows the accomplishment of many test variants at a relatively low cost. Therefore, it is possible to indicate the essential areas for further experiments or change the research programme. However, during the realisation of numerical simulations in terms of nonlinear mechanics, one should pay attention to a number of detailed problems associated with modelling of structures having key significance for the numerical modelling correctness.

The aim of the research presented herein was to check a possibility of numerical simulation through application of the Finite Elements Method in terms of of statical analises for reinforced concrete structures strengthened with CFRP. This work supplemented the program of experiments carried out at the Division of Concrete Structures, Faculty of Civil and Environmental Engineering, Koszalin University of Technology [4, 5].

## 2. Subject of research

The numerical research was planned in such a way that the verification of the calculation results versus the experimental results could be possible. The subject matter of the numerical analyses were tested reinforced concrete elements strengthened with CFRP [4].

The experiments were carried out on 12 reinforced concrete beams featuring the cross-section of 120×220 mm and length 3300 mm (Fig. 1), whereof 6 were strengthened with carbon fibre composite strips and 6 without strengthening. Two series of beams differing insignificantly in concrete strength properties such as (Table 1) modulus of elasticity (E), compression strength ($f_c$) and tensile strength ($f_t$) were used in the experiment. The beams were reinforced with steel bars with modulus of elasticity of 201 GPa and yield point of 433 MPa. The beams were strengthened with CFRP of 1.2x50 mm and 1.2x80 mm cross-sections of two degrees of external strengthening. The CFRP modulus of elasticity was 168 GPa, whereas its tensile strength was 2500 MPa. The tensile strength of applied epoxy glue used for strip to the beam attachment was 27.85 MPa.

The reinforced concrete beams were subjected to bending as freely supported of 3.0 m span by two concentrated forces of the same values located at 1/3 span intervals (Fig. 2). At each of the stages of the research the support reactions, vertical displacement, strip and structural steel deformation, as well as values of width and propagation of cracks were measured.

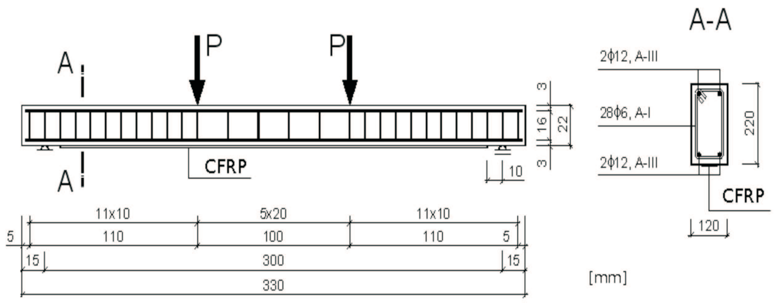



Fig. 1. Beam reinforcement and strengthening diagram.
Rys. 1. Schemat zbrojenia i wzmocnienia belek

**Table 1**

Primary strength properties of the concrete used in tests.
Podstawowe właściwości wytrzymałościowe zastosowanego w badaniach betonu

| Notation | E [GPa] | $f_c$ [MPa] | $f_t$ [MPa] |
|---|---|---|---|
| Concrete-I | 34.1 | 57.6 | 4.4 |
| Concrete-II | 36.6 | 51.6 | 3.7 |

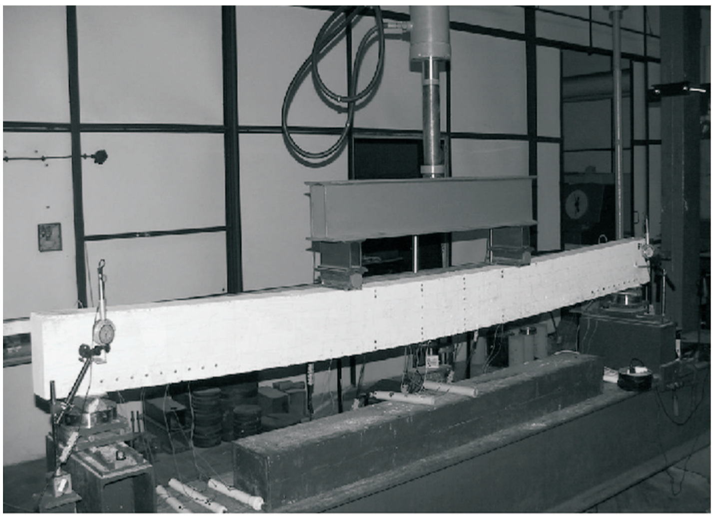


Fig. 2. Test stand view.
Rys. 2. Widok stanowiska badań eksperymentalnych

## 3. Numerical model and analysis

The Finite Elements Method (MES) was used as the method for numerical solution of the static problem of researched reinforced concrete structures.

In order to limit the size of numerical model, the ¼ real structure model, geometrically equivalent to the tested beams, was used in the analysis. The symmetry with relation to the vertical plane through the centre of beam span, and the symmetry with relation to the vertical plane through the beam horizontal axis was used.

The numerical model of reinforced concrete beams strengthened with CFRP was constructed of several type finite elements (Fig. 3). In a part of the reinforced concrete structure, three-dimensional solid elements of 8 nodes and 24 degrees of freedom were used. The reinforcement rods were discretized with spatial truss elements of 2 nodes and 6 degrees of freedom, whereas the CFRP strips were modelled by using three-dimensional shell elements of 4 nodes and 24 degrees of freedom with integration at 5 levels of thickness, using the Gauss method. In all types of elements, the linear shape functions were applied, whereas in the brick and shell elements the integration with hourglass control was additionally reduced in order to ensure compatibility of element description with the applied method of problem solution, and to eliminate any instability associated with the so-called "zero energy modes" [2].

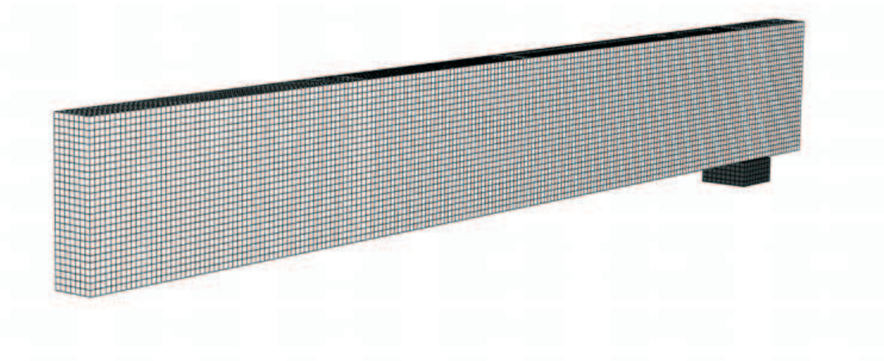
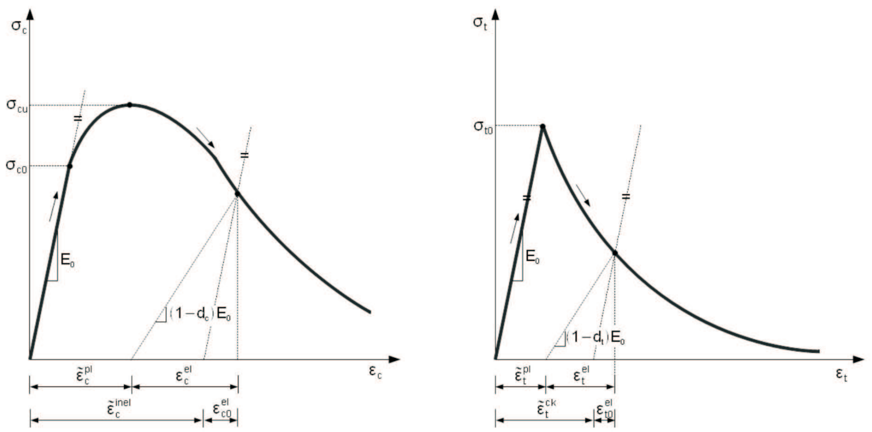


Fig. 3. Finite element mesh.
Rys. 3. Siatka elementów skończonych

The reinforcement and concrete cooperation was taken into account by application of the embedded element technique, whereas the CFRP and the cooperation of the reinforced concrete beam was modelled by application of the permanent tying of the strip elements mesh with concrete elements option [6]. Modelling of glued up strip to the beam attachment was omitted due to glue strength, higher than concrete tensile strength and strip detachment forms observed during the experiments, in which the tearing off was occurring with the detachment of a concrete layer [4].

Taking into account the used structure symmetry and real support method, boundary conditions were implemented in the model. As the typical mechanisms of defining the support conditions would lead, in this case, to an additional, strongly localized tension of beam elements associated with beam tearing off from its support, which did not happen in the experiment, a decision was taken to model the supports by high rigidity elements and to apply description of the master-slave type contact between the support and the beam, taking the friction into consideration [6].

A particularly important issue in solving such type of problems is to provide a possibly accurate modelling of properties of the material the structure is made of. In order to describe concrete material properties, the concrete damaged plasticity model developed by J. LUBLINER *et al.* [7, 8] was applied. This model takes into account the plasticity based damage response of concrete under compression, and the elastic and quasi-brittle concrete response in tension conditions (Fig. 4). In both loading conditions, the development of damage and degradation of strains occurring during the failure process with use of separate scalar damage parameters ($d_c$ – in compression, $d_t$ – in tension) was taken into account. At the same time, this took into consideration (at the same time) the yield criterion based on Drucker-Prager hyperbolic function and the rule of non-associated potential plastic flow. The reinforcing steel properties were described as elasto-plastic with Huber-Mises-Hencky plasticity condition, associating the potential rule of plastic flow and isotropic hardening. In the case of CFRP strips, considering their location and loading method, a linear-elastic description is sufficient.




a) b)

Fig. 4. Stress-strain relationships applied in the concrete material model: a) compression, b) tension.
Rys. 4. Zależności odkształcenie-naprężenie wykorzystywane w modelu materiałowym betonu: a) ściskanie, b) rozciąganie

Before the above-mentioned material descriptions were applied, it was essential to define and verify all required parameters of the models used. Initial comparative studies, in which compatibility of the numerical and experimental results at the stage of definition of the primary mechanical features, particularly in concrete, were very useful.

As it was assumed, the range of values of the load applied to the structure would be compatible with the range applied in the experiment (loading until destruction);

the concentrated forces were replaced with a pressure distributed around a small area. Such methodology was enforced by susceptibility of the adopted material description to considerable amounts of concentrated loads applied at nodes.

Geometric nonlinearity (large displacements) were considered in the model.

The finite elements mesh applied in the analysis comprised in total 37 345 finite elements with the total number of degrees of freedom being almost 137 K.

The complex numerical model presented herein faces problems in calculation, as regards obtaining of convergence during solving the numerical problem with the application of typical incremental-iterative methods. Therefore, the problem solving strategy based on *quasi-static* calculations was adopted here [2]; in such a case, the solution of nonlinear dynamic equations was effected by application of the explicit central difference integration rule. Due to conditional stability of the method and small time increment, it was essential and possible to apply in the hereby presented model the so-called mass scaling. The ABAQUS/Explicit software (Simulia, Inc.) was used in direct calculations [6].

During the numerical analysis, calculations for 6 variants of numerical model were performed:

- ◇ b1bw – Concrete-I beam without strengthening,
- ◇ b1wt5 – Concrete-I beam strengthened with 5 cm wide CFRP strip,
- ◇ b1wt8 – Concrete-I beam strengthened with 8 cm wide CFRP strip,
- ◇ b2bw – Concrete-II beam without strengthening,
- ◇ b2wt5 – Concrete-II beam strengthened with 5 cm wide CFRP strip,
- ◇ b2wt8 – Concrete-II beam strengthened with 8 cm wide CFRP strip.

In order to control correctness of the calculation, the attention was paid to the run of selected energy forms, among others, kinetic energy, hourglass control energy, plastic deformation dissipated energy, and energy released during brittle cracking.

## 4. Calculation results

In the obtained calculation results, main attention was focused on the status of displacements and damage to the analysed beams.

In Figs. 5 and 6, results of calculation pertaining to the course of the relationships between the maximum beam deflection and full load values, as well as the results recorded during the experiments, were shown. The b2wt8 and b2wt5 variants were not used in the experiments. A good compatibility of the calculated displacement values with those measured is noticeable. Significant differences between the calculated and measured values began to appear only in the case of arriving at the critical (destructive) load value. Similarity between the calculated and measured moment of occurrence of the first flaws (cracks perpendicular to element axis) can additionally be seen in the figures. This pertains to load values between 9.5 kN and 12 kN, depending on the beam variant.

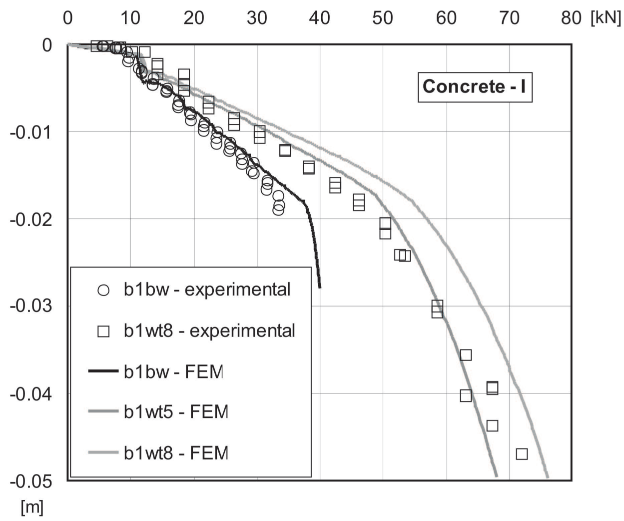



Fig. 5. Beam maximum deflection versus total load value relationships (Concrete-I).
Rys. 5. Zależności maksymalnego ugięcia belek od wartości całkowitego obciążenia (Concrete-I)

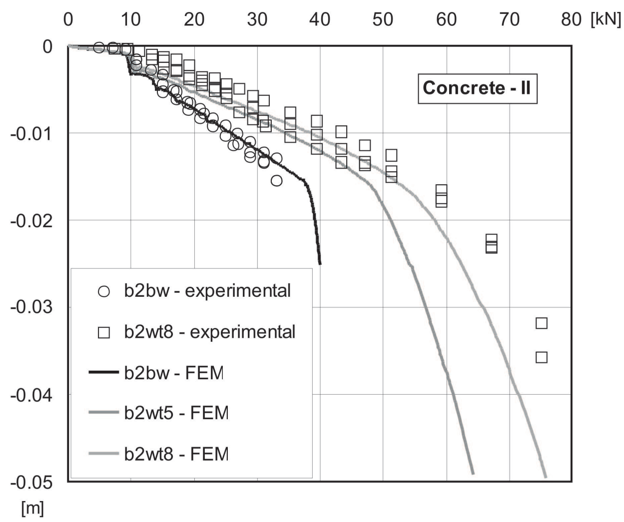



Fig. 6. Beam maximum deflection versus total load value relationships (Concrete-II).
Rys. 6. Zależności maksymalnego ugięcia belek od wartości całkowitego obciążenia (Concrete-II)

Figures 7, 8 and 9 show the selected distribution of damage observed in the numerical analysis and in experiments. Figure 6 shows distribution of damage in the beams at the initial stage. Similarity between the numerical and experimental values, both in terms of load value, which initiated the first damage, and in location of the damage, is seen. The first damage developed in the zone of constant bending moment in the form of a crack perpendicular to beam longitudinal axis. As the load values increased, the damage area expanded and the general trend of occurrence of further damage and their development was also concurrent with the experiment. Apart from perpendicular cracks, also diagonal cracks developed in the strengthened beams at the support sections thereof. Figures 7 and 8 show a comparison of selected numerical analyses and experimental results. For the majority of calculation variants, the number of damaged elements at the same load values is higher in the case of the strengthened beams than in the non-strengthened ones. This is compatible with the experimental results and consist in development of a higher number of cracks featuring smaller width values than those in non-strengthened beams.

a)
b)
c)
d)

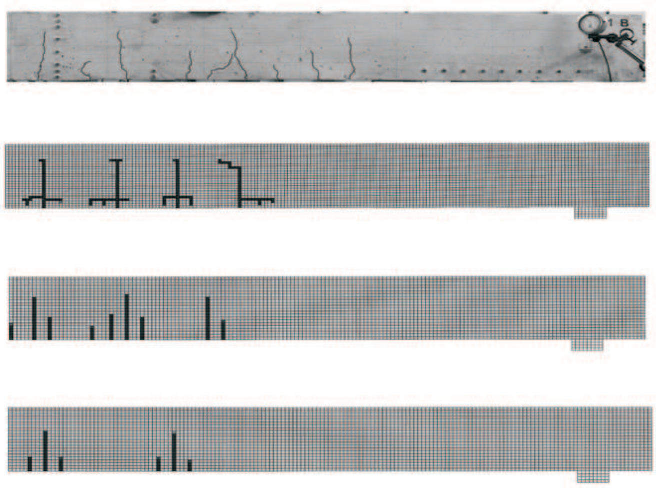


Fig. 7. Distribution of initial beam damage (at 12 kN load): a) experimental result, b), c), d) numerical results.
Rys. 7. Rozkład początkowych uszkodzeń w belkach (obciążenie 12 kN): a) wynik doświadczalny, b), c), d) rezultaty numeryczne

Of course, in the applied numerical model it is impossible to find out the values of crack width because imaging of the damage is in this case associated with the size of the adopted finite elements, however, some concurrence in damage density can be noted – it is lower in the non-strengthened model and higher in the strengthened one.

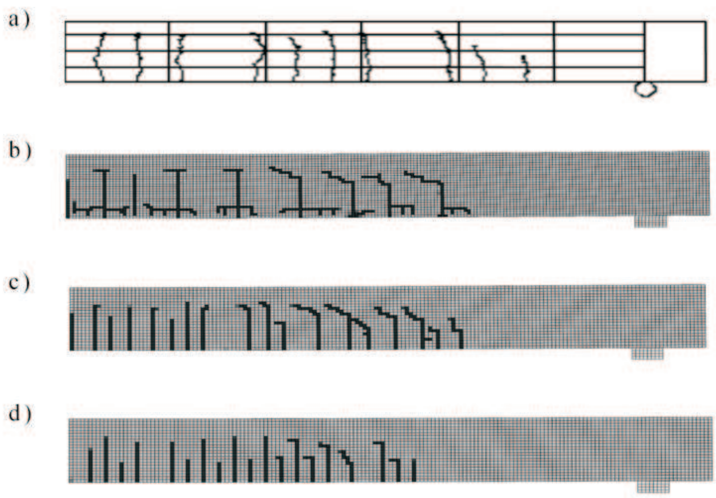


Fig. 8. Distribution of beam damage (at 24 kN load): a) experimental result, b), c), d) numerical results.
Rys. 8. Rozkład początkowych uszkodzeń w belkach (obciążenie 24 kN): a) wynik doświadczalny, b), c), d) rezultaty numeryczne

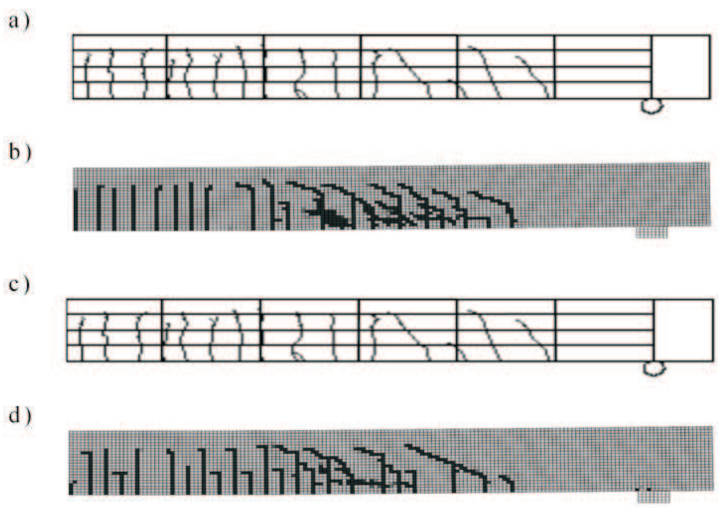


Fig. 9. Distribution of beam damage (at 47 kN load): a), c) experimental results, b), d) numerical results.
Rys. 9. Rozkład początkowych uszkodzeń w belkach (obciążenie 47 kN): a), c) wynik doświadczalny, b), d) rezultaty numeryczne

## 5. Summary

Based on the performed numerical analyses of reinforced concrete beams strengthened with carbon fibre reinforced polymer strips and comparison of the results with the experimental values, it may be observed that the FEM analysis allows to obtain good results within the range of determined displacements.

When the beam damage condition was compared, initially the similarity of the results regarding the moment of occurrence of the first damage (cracks) and their initial development was observed, however, as the load value was approaching the destruction value, the similarity deteriorated as the load value was approaching the destruction value. This was evidently associated with problems in the description of the destruction process.

A difficult problem in the analysis of numerical performance lies in the application of a proper concrete material model and proper identification of all required model parameters.

As it appears from the numerical and experimental results pertaining to the beams of a varied degree of external strengthening, the effect of reinforced concrete beam strengthening increases with the increase of CFRP transversal cross-section due to deflection, though its impact is insignificant.

Due to the strongly nonlinear character of the analysed process and performance of the analysis until destruction, it would be beneficial to apply the quasi-static calculations strategy with the so-called mass scaling, but a drawback of such approach is a considerable cost of calculation causing a necessity for fairly long waiting for analysis result, even when efficient computer systems are used.

It is important to note that numerical analysis of complex reinforced concrete structures strengthened with composite material can result in good effects. The experiment and calculations complement each other, which allows for mutual benefits. On one hand, the experiment allows the verification of the adopted theoretical model through comparison of the calculation and experimental results. It also defines the limits for confidence in the numerical analysis results. This allows the development of methods for more efficient physical processes modelling. On the other hand, numerical studies allow to accomplish computer simulations for various variants of structure, its strengthening, or various impacts and their combinations, thus providing a great number of results, which would be difficult to record in an experiment.

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## Extended abstract

Development of composite materials causes that more and more frequently structures made of traditional and composite materials become a solution for engineering problems pertaining to, among other things, strengthening of building structures.

The aim of the research work presented herein was to check a possibility of numerical simulation in terms of accomplishment of static analysis for reinforced concrete structures strengthened with carbon fibre reinforced polymer strips. This research work supplemented the program of experiments carried out at the Division of Concrete Structures at the Faculty of Civil and Environmental Engineering, Koszalin University of Technology.

Reinforced concrete beams strengthened with composite strips (CFRP) made of carbon fibres and epoxy resin featuring various width values as well as non-strengthened bent beams were analysed. The beams arranged in a free support scheme were being bent by two concentrated forces within full range of loading (until destroyed).

The numerical analysis was performed trough application of the Finite Elements Method (FEM) and the calculation model applied took into account the geometric and physical non-linearity, the beam to support contact problem and cooperation of reinforced concrete and composite strip. One of the most important elements of the job was proper adoption of concrete material model. The damaged plasticity model for concrete taking into account the plasticity-based damage response of concrete in compression and elastic and quasi-brittle concrete response in tensile conditions was applied in the calculations. Model parameters were selected based on the experimental test results for primary concrete strength parameters.

The problem has been solved by application of the *quasi-static* strategy of calculations with central-difference method using ABAQUS software.

Attention was focused on, in the results obtained, on displacements and reinforced concrete cracks development as well as on the beam destruction process. Response of beams strengthened with CFRP strips of two width values and of non-strengthened beams was analysed. Good concurrence of the experimental and numerical results were obtained in the majority of cases.

## ANALIZA STATYCZNA BELEK ŻELBETOWYCH WZMOCNIONYCH TAŚMAMI KOMPOZYTOWYMI

### Streszczenie

Rozwój w zakresie materiałów kompozytowych powoduje, że coraz częściej konstrukcje złożone z tradycyjnych i kompozytowych materiałów stają się rozwiązaniem problemów inżynierskich dotyczących m.in. wzmacniania konstrukcji budowlanych.

Celem prezentowanych badań było sprawdzenie możliwości symulacji numerycznej w zakresie analizy statycznej konstrukcji żelbetowych wzmacnianych taśmami kompozytowymi. Badania te stanowiły uzupełnienie programu doświadczeń przeprowadzonych w Katedrze Konstrukcji Betonowych na Wydziale Budownictwa i Inżynierii Środowiska Politechniki Koszalińskiej.

W badaniach analizowano zginane belki żelbetowe z wzmocnieniem w postaci taśm kompozytowych (CFRP) z włókien węglowych i żywicy epoksydowej o różnej szerokości oraz bez wzmocnienia. Belki w swobodnie podpartym układzie statycznym były zginane przez dwie siły skupione w pełnym zakresie obciążeń (aż do zniszczenia).

Numeryczną analizę statyczną przeprowadzono stosując Metodę Elementów Skończonych (MES), a zastosowany model obliczeniowy uwzględniał nieliniowości geometryczne i fizyczne, zagadnienie kontaktu pomiędzy belką i podporami oraz współpracę betonu ze zbrojeniem i taśmą kompozytową. Jednym z najważniejszych elementów zadania było właściwe przyjęcie modelu materiałowego betonu. W obliczeniach przyjęto sprężysto-plastyczny model betonu ze zniszczeniem uwzględniający sprężysto-plastyczno-kruchy charakter pracy betonu przy ściskaniu oraz sprężysto-kruche zachowanie przy rozciąganiu. Parametry modelu zostały dobrane na podstawie wyników badań doświadczalnych podstawowych parametrów wytrzymałościowych betonu.

Zagadnienie zostało rozwiązane przy zastosowaniu quasi-statycznej strategii obliczeń z wykorzystaniem procedury różnic centralnych (oprogramowanie ABAQUS).

W uzyskanych wynikach zwrócono uwagę na rozwój przemieszczeń oraz zarysowania belek żelbetowych oraz na proces zniszczenia belek. Analizowano odpowiedź belek wzmocnionych taśmami o dwóch szerokościach oraz belki bez wzmocnienia. W większości analiz uzyskano dobrą zbieżność pomiędzy wynikami badań doświadczalnych i numerycznych.

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