ECCM-2001

European Conference on
Computational Mechanics

June 26-29, 2001
Cracow, Poland

NUMERICAL ANALYSIS OF CONCRETE SLAB SUBJECTED TO BLAST LOADING

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Key words: Concrete, Material modelling, Blast loading

Abstract. *The paper concerns the application of rate-dependent plastic-damage constitutive model for concrete into a non-linear numerical analysis of reinforced concrete slab subjected to the impulsive load produced by a contact explosion. Main theoretical assumptions adopted in this analysis have been practically examined and discussed, in order to provide an efficient material modelling for concrete structures analysed with the use of commercial finite element codes. The adequate computer simulations of experimental tests performed by D. Kraus at al. [1] have been realised, checking the validity and applicability of assumed material model for concrete*

1 Introduction

The numerical simulation of structural response for the blast load produced by explosive charge is currently a problem of growing importance in many practical situations, no longer limited to military applications. Therefore, the possibility of concrete response modelling in the case of short duration, high amplitude loads is a problem of vital importance in the design of structures subjected to this kind of load [2, 3].

In the present work, the highly non-linear shock-wave phenomena in reinforced concrete slab loaded by a contact explosion are analysed. This problem has been studied by many researchers, the most up to date review of publications one can find in Proceedings of the International Conference on Computational Modelling of Concrete Structures „Euro-C". In the published works concerning problems mentioned above, various aspects of analysis are taken into considerations: material modelling, discretization of structure, numerical modelling of blast load, description of eventual contact, as well as the numerical algorithms applied in order to solve the entire problem in time and space domain.

The numerical analysis presented in this paper is based on the experimental results obtained by D. Kraus at al. [1], where the explosive masses of 0.5 and 1.0 kg of PETN have been detonated on the surface of reinforced slabs 2.0x2.0x0.3 m. The final results (i.e. generated craters, their dimensions and shape) are also studied, as well as the material status in the slab interior.

In order to obtain the reliable results of a numerical analysis, the rate - dependent plastic - damage constitutive model has been applied, based on considerations presented by R.Faria and X.Oliver in [4]. The damage model is defined by two independent internal damage variables, in order to characterise the non-linear mechanism of degradation of concrete under tensile or compressive loading conditions.

For the reinforcement, the classic von Mises elasto-plastic formulation for steel has been adopted, with rate-dependent assumption according Cowper-Symonds [5].

The assumed material model has been implemented into the computer code Abaqus/Explicit Ver. 5.8 [5], as the user subroutine. Detailed description of the numerical realisation of this algorithm and the adequate flowchart is given by authors in [6]. The remarks and various considerations about problems of a practical application of the material model considered here are also provided there.

2 Material model for concrete

In the present work, an elasto-plastic model with damage [4] has been selected, with two independent internal damage variables, in order to characterise the independent nonlinear mechanisms of degradation of concrete, under tension or compression. This provides a constitutive formulation with capability of describing the overall nonlinear stress-strain curves, including the strain-softening response, and the stiffness degradation mechanism.

Generally, the model adopted here, has its basis in the continuum damage mechanics, firstly introduced by Kachanov [8].

An expression for the effective stress tensor has assumed the following form:

$$\bar{\sigma} = \mathbf{D}_0 : \left(\varepsilon - \varepsilon^{\mathrm{p}}\right) \tag{1}$$

where $\mathbf{D_0}$ is the fourth order linear-elastic constitutive matrix, ε is the second order strain tensor, ε^p is the plastic strain tensor.

The following form of Helmholtz free energy potential has been adopted:

$$\Psi(\varepsilon, \varepsilon^{\mathrm{p}}, \mathrm{d}^+, \mathrm{d}^-) = (1 - \mathrm{d}^+)\Psi_0^+(\varepsilon, \varepsilon^{\mathrm{p}}) + (1 - \mathrm{d}^-)\Psi_0^-(\varepsilon, \varepsilon^{\mathrm{p}}) \tag{2}$$

The elastic free energies are defined as follows:

$$\begin{aligned} \Psi_0^+ &= \frac{1}{2}\bar{\sigma}^+ : \mathbf{D}_0^{-1} : \bar{\sigma} \\ \Psi_0^- &= \frac{1}{2}\bar{\sigma}^- : \mathbf{D}_0^{-1} : \bar{\sigma} \end{aligned} \tag{3}$$

$\mathrm{d}^+, \mathrm{d}^-$ are the damage variables, assigned relatively to compression and tension. Their values are of the range <0,1>.

In order to characterise the damage a concept of effective compressive and tensile stress has been introduced. In the present work they will assume the following forms:

$$\begin{aligned} \bar{\mathrm{r}}^+ &= \sqrt{\bar{\sigma}^+ : \mathrm{D}_0^{-1} : \bar{\sigma}^+} \\ \bar{\mathrm{r}}^- &= \sqrt{\sqrt{3}(\mathrm{K}\bar{\sigma}_{\mathrm{oct}}^- + \bar{\tau}_{\mathrm{oct}}^-)} \end{aligned} \tag{4}$$

K is a material property, and the $\bar{\sigma}_{\mathrm{oct}}^-, \bar{\tau}_{\mathrm{oct}}^-$ are the octahedral normal stress and the octahedral shear stress, calculated from $\bar{\sigma}^-$.

On the basis of calculated effective equivalent stresses, two damage criteria has been introduced, according to J.Simo and J. Ju [9]:

$$\begin{aligned} \mathrm{g}^+(\bar{\mathrm{r}}^+, \mathrm{r}^+) &= \bar{\mathrm{r}}^+ - \mathrm{r}^+ \leq 0 \\ \mathrm{g}^-(\bar{\mathrm{r}}^-, \mathrm{r}^-) &= \bar{\mathrm{r}}^- - \mathrm{r}^- \leq 0 \end{aligned} \tag{5}$$

where the entities r^+ and r^- are current damage thresholds, and control the size of damage surfaces.

The rates of tensile and compressive variable are defined as follows:

$$\begin{aligned} \dot{\mathrm{d}}^+ &= \dot{\bar{\mathrm{r}}}^+ \frac{\partial \mathrm{G}^+(\bar{\mathrm{r}}^+)}{\partial \bar{\mathrm{r}}^+} = \dot{\mathrm{G}}^+ \geq 0 \\ \dot{\mathrm{d}}^- &= \dot{\bar{\mathrm{r}}}^- \frac{\partial \mathrm{G}^-(\bar{\mathrm{r}}^-)}{\partial \bar{\mathrm{r}}^-} = \dot{\mathrm{G}}^- \geq 0 \end{aligned} \tag{6}$$

G^+, G^- are the appropriate monotonically increasing functions, in order to obtain the values of damage variables in the range of <0,1>.

Similarly to the damage variables, the rate of plastic strain tensor has been introduced in the form of:

$$\dot{\varepsilon}^{p} = \beta E H(\dot{d}^{-}) \frac{<\overline{\sigma}:\dot{\varepsilon}>}{\overline{\sigma}:\overline{\sigma}} \mathbf{D}_0^{-1}:\overline{\sigma} \tag{7}$$

where E is a Young modulus, β is the parameter that controls the rate intensity of plastic deformation. H is the Heaviside function of compressive damage rate, introduced to cancel plastic evolution during unloading.
After the splitting of the effective stress tensor $\overline{\sigma}$ into adequate tension and compression contributions $\overline{\sigma}^{+}, \overline{\sigma}^{-}$, the Cauchy stress tensor can be defined:

$$\sigma = (1 - d^{+})\overline{\sigma}^{+} + (1 - d^{-})\overline{\sigma}^{-} \tag{8}$$

The detailed description of the assumed algorithm, together with necessary remarks concerning its numerical application can be found in [10].

3 Description of experiment

The experiment was described by D. Kraus at al. in [1]. The test specimen consists of a slab with the dimensions 2.0 x 2.0 x 0.3 m, reinforced with 16 mm bars in a distance of 150 mm, localised near upper and bottom side of the slab. Concrete compressive strength (B35) $f_{ck} = 40$ MN/m^2. Reinforcement is of steel BSt 500/550, with strength f_{cs}=500 MN/m^2.
Explosive masses of 0.5 and 1.0 kg PETN with a density of 1.5 g/cm^3 were used. The charge was formed as a cube, positioned at the centre of the slab. Two different cases of explosion were examined: without any confinement and confined (the explosive charge was covered by sand bags).

As the results of the experiment, the dimensions of craters formed during the explosions have been presented. The entire experiment was also numerically simulated by its authors, using the Autodyn [6] computer code. In this code, the coupled Euler-Lagrange formulation based on the finite difference method is adopted. Because of this, it is possible to discretise the entire physical system: air-charge-structure. In order to model the nonlinear behaviour of concrete, the Drucker-Prager elasto-plastic model has been assumed.

4 Numerical simulation of the experiment

In order to analyse the practical problem described above, the discrete finite element model was built in environment of Abaqus/Explicit computer code. Because of a symmetry of the problem, one fourth of the structure has been analysed, with adequate boundary conditions. Three-dimensional brick elements with one point of Gauss integration were applied: 40 x 40 x 12 elements. This guarantees the regular shape of finite elements. Reinforcement was modelled as the rebar layers introduced into adequate sets of brick elements.

Load produced by the explosion was simulated as a field of pressure, variable in space and time, according to equations given by J. Henrych [7]. In this description, the value of maximum incident pressures Δp_ϕ at the distance R [m] from the centre of TNT charge W [kg] depends on the parameter:

$$\overline{R} = \frac{R}{\sqrt[3]{W}}; \tag{9}$$

according to the following equations:

$$\begin{aligned}
\Delta p_\phi &= \frac{14.0717}{\overline{R}} + \frac{5.5397}{\overline{R}^2} - \frac{0.3572}{\overline{R}^3} + \frac{0.00625}{\overline{R}^4}; \left[\mathrm{kp/cm^2}\right], && 0.05 \le \overline{R} \le 0.3 \\
\Delta p_\phi &= \frac{6.1938}{\overline{R}} - \frac{0.3262}{\overline{R}^2} + \frac{2.1324}{\overline{R}^3}; \left[\mathrm{kp/cm^2}\right], && 0.3 \le \overline{R} \le 1 \\
\Delta p_\phi &= \frac{0.662}{\overline{R}} + \frac{4.05}{\overline{R}^2} + \frac{3.288}{\overline{R}^3}; \left[\mathrm{kp/cm^2}\right], && 1 \le \overline{R} \le 10
\end{aligned} \tag{10}$$

In order to model the contact explosion (i.e. confined from one side), and to simulate the presence of the PETN charge (different from TNT), the adequate correction parameters have been applied, according to considerations given in [7].

Additionally, the duration of overpressure according to Henrych is given by a simple formula:

$$t_{ovp} = \sqrt[3]{W} * 10^{-3}(0.107 + 0.444\overline{R} + 0.264\overline{R}^2 - 0.129\overline{R}^3 + 0.0336\overline{R}^4) \tag{11}$$

The entire set of functions enables the user to describe the change of overpressure with time (for the established point in space):

$$\Delta p(t) = \Delta p_\Phi (1 - \frac{t}{t_{ovp}}) \tag{12}$$

This simple formulation was adopted into Abaqus/Explicit in a form of user subroutine, calculating the values of an overpressure according to equations (10 - 12).

5 Discussion of results

From the practical point of view, the most interesting problem is the final state of structure, i.e. its permanent deformation, extent of damaged zones, and amount of energy, dissipated on plastic deformations. This information gives the user a possibility to evaluate the structural strength, necessity of eventual repairs, or limits in exploitation of the structure.

In order to compare numerical results with experimental outcome, only the case of 1 kg PETN charge has been analysed here. The numerical analysis was performed for the assumed period of time 50 ms. This assures the termination of all plastic and damage effects, and gives the user a final set of results.

In Fig. 1 the final contour plot of vertical displacements (direction 3) is presented. The maximum displacement is localised just under the centre of explosion, and vary with thickness. All the values of displacements are relatively small, because the stiffness of the structure is very high, and the damage effects prevail.

A contour plot of plastic equivalent strain is displayed in Fig.2. The maximum effects in terms of plastic deformation are localised under the centre of the slab, on its upper and bottom surface. It coincides with the maximum damages, depicted in Fig. 3. In this figure, the damaged elements are removed from the mesh. The comparison with experimental results [1]. i.e. dimensions of craters, gives a good approximation of final geometry of the slab. Another zone of damages is localised at the end of the slab, near supports. The relatively high, but limited in space, values of equivalent plastic strains are due to supports reaction to the blast load.

The functions of energies versus time are given in Fig. 4. As one can find, all the dissipate effects terminate after approximately 10 ms. After this time, only elastic vibrations will occur.

6 Conclusions

The conclusions arising from this study indicate the great meaning of assumed material model in terms of obtained results of numerical analysis performed by means of a commercial finite element computer code. From a practical point of view, to obtain the necessary information regarding the state of the structure subjected to an explosion, the material model assumed in numerical analysis should be able to describe complex behaviour of material, from pure elastic response up to consequent degradation and damage. Although all phenomena which occur in material during dynamic response are very complex and coupled, to obtain the reasonable useful results of numerical analysis the relatively simple models, based on the assumed elasto-plastic behaviour of material, and taking into account the damage of material as well as its rate-dependent features, should be implemented into existing finite element codes.

In the case considered in this paper, the relatively simple formulation of material model for concrete, together with very simple elasto-plastic model for reinforcing steel, give a sufficiently precise results, in terms of final state of damages in the structure. The moderate time of analysis is also an important profitable feature, in comparison with other, more complicated material models.

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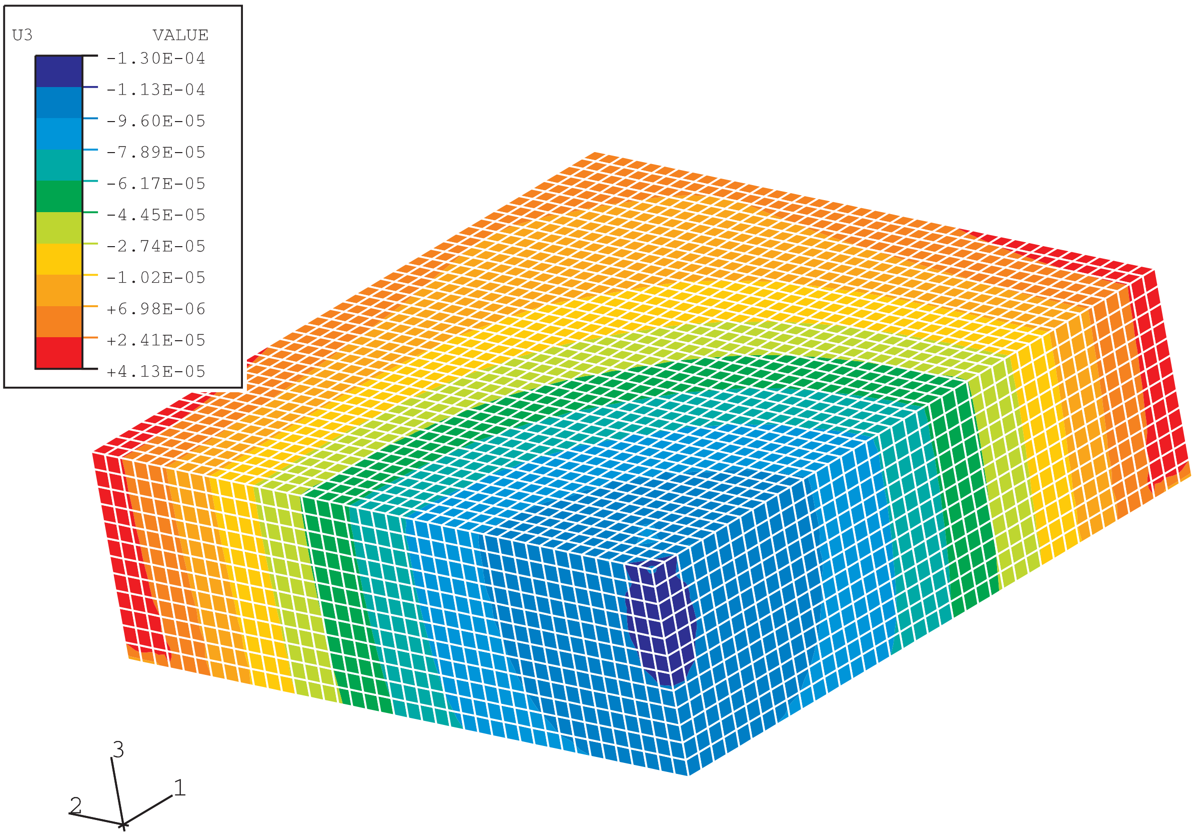

Fig.1. Vertical displacement

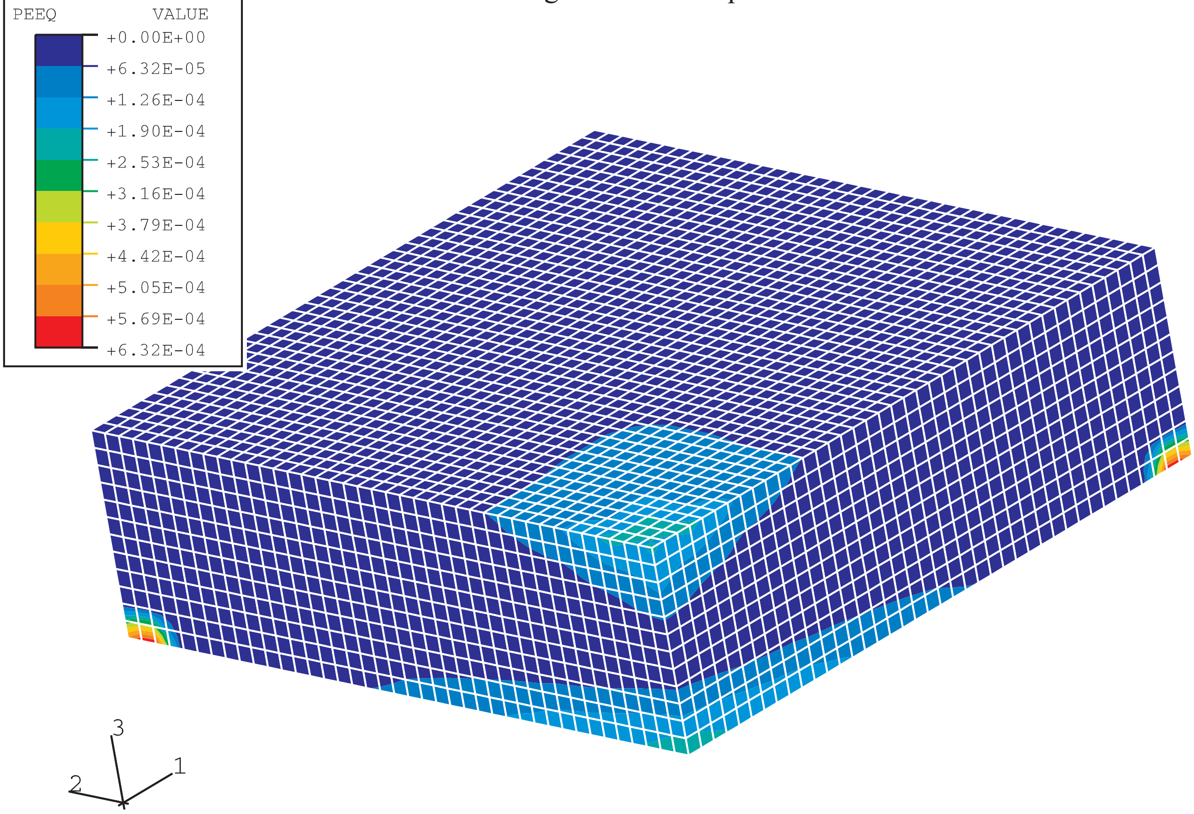

Fig.2. Plastic equivalent strain

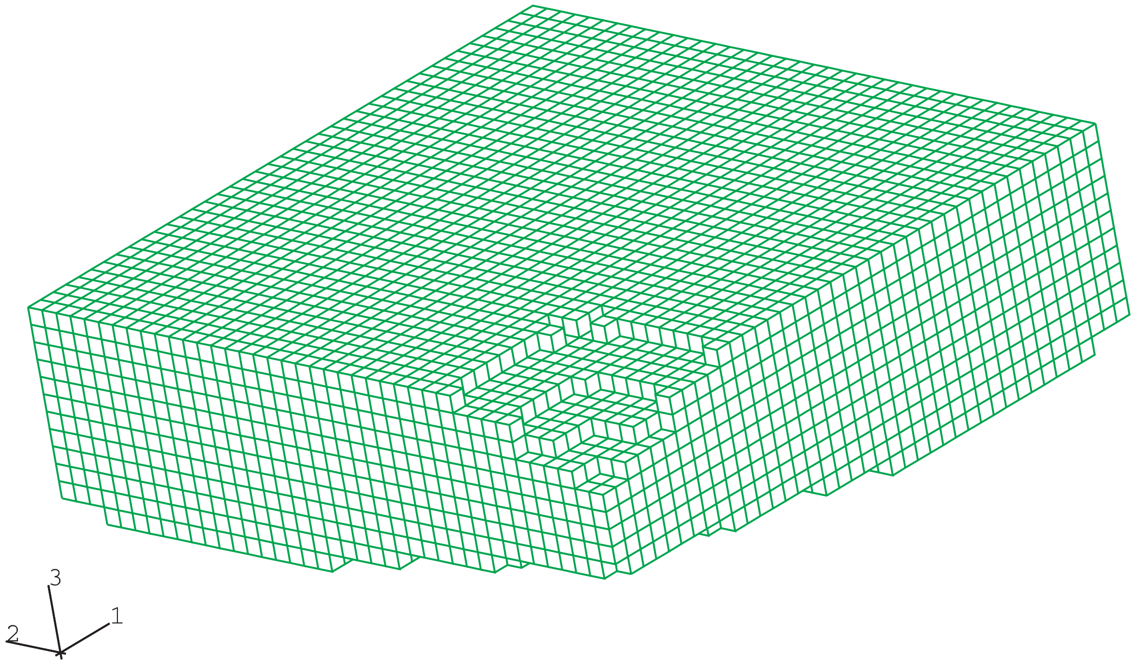

Fig.3. Final configuration of slab

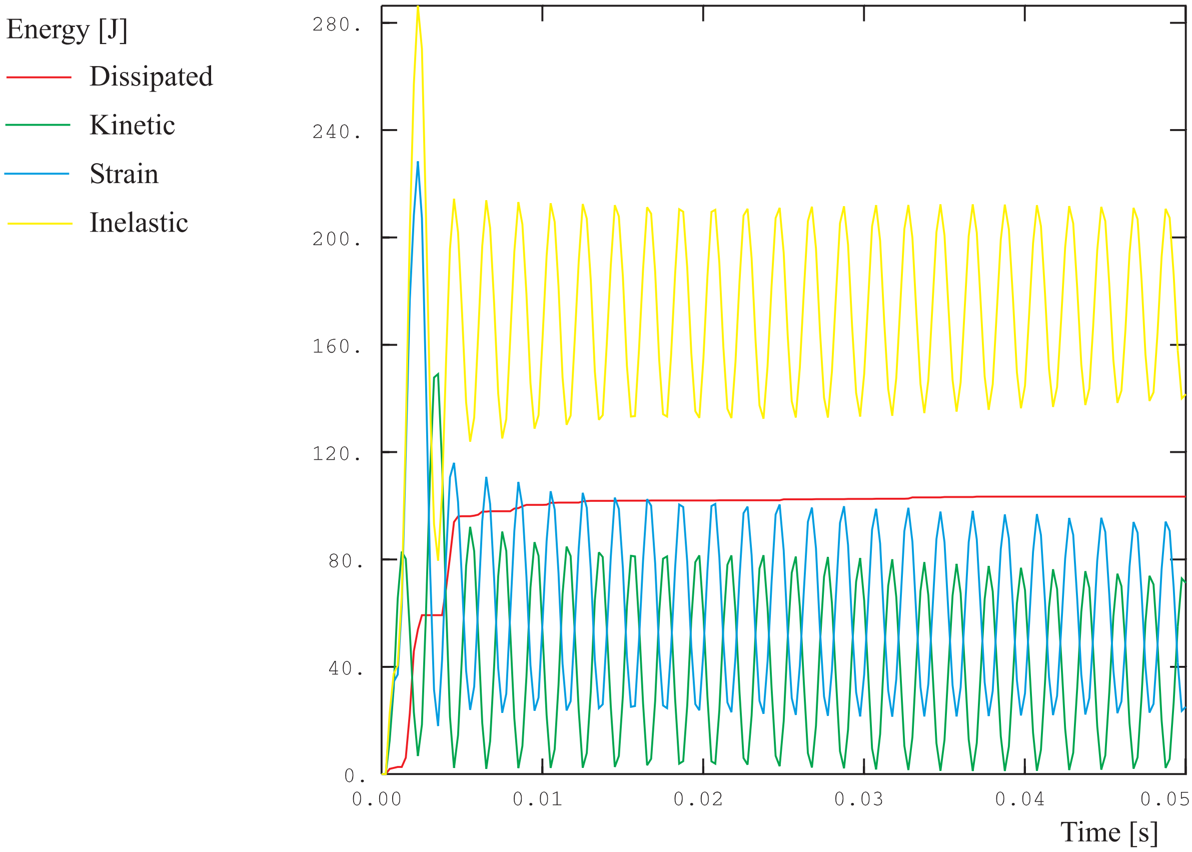

Fig.4. Energies versus time